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BY

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ABSTRACT

An expression for the self-impedance is derived for any size of baffle with piston sizes in the range $ka = 0.75 - 2$. The unbaffled condition, as a particular case, provides a means of checking the expression against the few existing results derived by more advanced methods and good agreement is obtained. Certain sizes of finite baffle raise the self resistance and reactance somewhat above the infinitely baffled values.

INTRODUCTION

Values for the acoustic self-impedance of a rigid circular piston source in an infinite baffle are generally available in the literature. The method given here for the cases where the baffle is finite or entirely absent uses the above-mentioned values in conjunction with a principle of superposition. This assumes that the acoustic pressure experienced by the piston when infinitely baffled has two components; namely, the pressure it would experience if the baffle were absent, and that pressure which would result if the baffle were replaced by an infinite (non-uniform) source which at every point on it creates a pressure equal to that actually experienced there by the baffle. If this latter component is evaluated and deducted from the known values of pressure on the piston when infinitely baffled, then the unbaffled values of pressure, and thence self-impedance, will remain.

The replacement of the infinite baffle by a non-uniform source follows, of course, the method of treating problems of reflexion from a rigid wall, where the normal velocity is made to vanish by an appropriate source strength at each point. The present case is rather different in that, since the piston source and infinite baffle are coplanar, there is no incident wave in the sense of one having a normal velocity component. The strength of the fictitious source at each point is therefore determined by the (scalar) pressure occurring at the corresponding point on the infinite baffle.

The case of a finite baffle is treated similarly. Thus, if the finite baffle has outer radius b and inner radius a (the piston radius), the radiation pressure on the piston is assumed to be given by deducting from the infinitely-baffled value a component due to a non-uniform source whose inner radius is b and which extends to infinity. The procedure for dealing with a finite baffle does not take account of diffraction of the sound behind the baffle, nor the thickness of the baffle, and in this respect is an approximation. On the other hand, the application of the method is intended primarily to situations where limited baffling occurs by virtue of the presence of adjacent equipment, frameworks, structures and the like, none of which can usually be reasonably represented by a thin rigid baffle.

2. THE VELOCITY POTENTIAL IN THE PLANE OF AN INFINITE BAFFLE

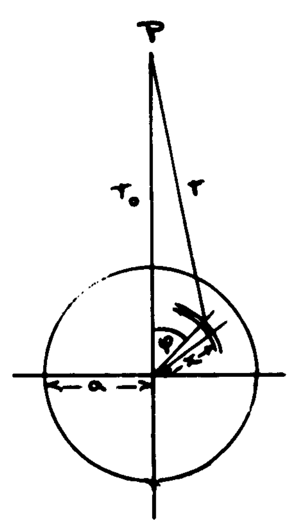

FIG. No. 1

Let ψ_1 be the velocity potential at a point P in the plane of the infinite baffle. Then if a is the piston radius and r_o is the distance of P from the piston centre (see Fig. No. 1), the potential is given by -

$$\psi_1 = -\frac{u}{\pi}\int_0^a x\,dx\int_0^\pi e^{-ikr}\,d\varphi \qquad (1)$$

- where u is the vibrational velocity of the piston

Since $r = \left(r_o^2 + x^2 - 2\,r_o x\cos\varphi\right)^{\frac{1}{2}}$ the first integral in (1) can be evaluated by expressing the integrand as a series of Bessel and Legendre functions [1] -

$$\frac{e^{-i\kappa r}}{r} = \frac{1}{i\,\kappa\, r_o x} \sum_{n=0}^{a} (2n+1)\, \hat{J}_n(\kappa x) \left\{ \hat{J}_n(\kappa r_o) - i\, \hat{N}_n(\kappa r_o) \right\} P_n(\cos\varphi) \qquad (2)$$

$$\text{- where } \hat{J}_n(x) = \left(\frac{\pi x}{2}\right)^{\frac{1}{2}} J_{n+\frac{1}{2}}(x) \text{ and } \hat{N}_n(x) = \left(\frac{\pi x}{2}\right)^{\frac{1}{2}} N_{n+\frac{1}{2}}(x)$$

The double integral then becomes -

$$\int_0^a x\,dx \int_0^{\pi} \frac{e^{-i\kappa r}}{r} = \frac{-1}{\kappa r_o} \sum_{n=0}^{\infty} (2n+1) \left\{ i\, \hat{J}_n(\kappa r_o) + \hat{N}_n(\kappa r_o) \right\} \int_0^a \hat{J}_n(\kappa x)\, dx \int_0^{\pi} P_n(\cos\varphi)\, d\varphi$$

This vanishes for all odd values of n. Also if κa does not exceed 2 there is very little error entailed by taking only the first two finite terms (i.e. $n = 0,2$), a device which greatly eases the integration problems met below in dealing with the non-uniform source.

$$\psi_1 = \frac{u}{\kappa^2 r_o} \left[\left\{ i\, \hat{J}_o(\kappa r_o) + \hat{N}_o(\kappa r_o) \right\} \int_0^{\kappa a} \hat{J}_o(s)\, ds + \frac{5}{4} \left\{ i\, \hat{J}_2(\kappa r_o) + \hat{N}_2(\kappa r_o) \right\} \int_0^{\kappa a} \hat{J}_2(s)\, ds \right] \qquad (3)$$

$$\text{since } \int_0^{\pi} P_o(\cos\varphi)\, d\varphi = \pi, \text{ and } \int_0^{\pi} P_2(\cos\varphi)\, d\varphi = \frac{\pi}{4}$$

3. THE NON-UNIFORM SOURCE

The infinite baffle is replaced entirely - or that part of it beyond some radius b, as explained above - by a non-uniform source. The source strength at any point is such as to balance the pressure actually experienced by the infinite baffle. If this is p_1 at any point then the required source velocity there can be given, under a limited range of conditions, fairly accurately by the plane-wave relation $v = p_1/\rho c$. The required velocity may be written in terms of ψ_1 ; $v = i\kappa\psi_1$. The range of conditions referred to obviously excludes any relatively rapid change of pressure along the baffle.

If then the potential at a point on the baffle due to the piston is ψ_1, the fictitious source velocity there is $i\kappa\psi_1$ and the velocity potential due in turn to this elementary source is -

$$\psi_{11} = \frac{-i\kappa\psi_1}{2\pi}\, e^{-\frac{i\kappa r}{r}}\, dA \qquad (4)$$

- where ψ_1 is expressed by (3), r is the distance from the elementary source and dA is its area. The pressure corresponding to (4) is then -

$$P_{11} = \frac{\rho\omega\kappa\psi_1}{2\pi} \, e^{-\frac{i\kappa r}{r}} \, dA \tag{5}$$

By integrating (5) over the piston, the force thereon due to the elementary source is obtained. thus -

$$df = \frac{\rho\pi\kappa\, dA}{\pi} \int_0^a x\, dx \int_0^\pi \psi_1 \cdot e^{-\frac{i\kappa r}{r}} \, d\varphi \tag{6}$$

referring again to Fig. 1, where P is now the elementary source. On account of the symmetry of the field about the piston axis, it is possible to consider the elementary source as an annulus having uniform source strength and whose radius and width are r_o and dr_o respectively (see Fig. 1). That is, $dA = 2\pi r_o dr_o$ in (6) and a third integration will yield the force on the piston due to a non-uniform source corresponding to an infinite baffle whose inner radius is, say, b . Thus -

$$f = 2\,\rho\omega\kappa \int_0^\infty r_o dr_o \int_0^a x\, dx \int_0^\pi \psi_1 \, e^{-\frac{i\kappa r}{r}} \, d\varphi \tag{7}$$

Introducing again the relationship expressed in (2)

$$\int_0^a x\, dx \int_0^\pi \psi_1 \, e^{-\frac{i\kappa r}{r}} \, d\varphi = \frac{\psi_1 \pi}{\kappa^2 r_o} \Big[\left\{ i\,\hat{J}_o(\kappa r_o) + \hat{N}_o(\kappa r_o) \right\} \int_0^{\kappa a} \hat{J}_o(s)\, ds + \tfrac{5}{4} \left\{ i\,\hat{J}_2(\kappa r_o) + \hat{N}_2(\kappa r_o) \right\} \int_0^{\kappa a} \hat{J}_2(s)\, ds \Big] =$$

$$= \frac{-\pi u}{\kappa^4 r_o^2} \Big[\left\{ i\hat{J}_o(\kappa r_o) + \hat{N}_o(\kappa r_o) \right\} \int_0^{\kappa a} \hat{J}_o(s) ds + \tfrac{5}{4} \left\{ i\hat{J}_2(\kappa r_o) + \hat{N}_2(\kappa r_o) \right\} \times \int_0^{\kappa a} \hat{J}_2(s) ds \Big]^2 \tag{8}$$

- where again only the first two finite terms in (2) are used. By discarding the term containing

$$\left(\int_0^{\kappa a} \hat{J}_2(s)\, ds \right)^2 ,$$

which is evidently a relatively very small quantity provided that $\kappa a \ngtr 2$, the third integral in (7) is easily evaluated and the total force on the piston due to the non-uniform source can be expressed as -

$$
f = \frac{2\pi\rho c u}{\kappa^2}\left[\left(\int_0^{\kappa a} \hat{J}_0(z)\, dz\right)^2 \left\{ \int_{2\kappa b}^{\infty} \frac{\cos}{\upsilon}\, \upsilon\, d\upsilon - i \int_{2\kappa b}^{\infty} \frac{\sin}{\upsilon}\, \upsilon\, d\upsilon \right\} + \right.
$$

$$
\left. + \frac{5}{4}\int_0^{\kappa a} \hat{J}_0(z)\, dz \left\{ \left| - \frac{6 \cos \upsilon}{\upsilon^2} \right|_{2\kappa b}^{\infty} - \int_{2\kappa b}^{\infty} \frac{\cos \upsilon}{\upsilon}\, d\upsilon + \left| i\, \frac{6 \sin \upsilon}{\upsilon^2} \right|_{2\kappa b}^{\infty} + i \int_{2\kappa b}^{\infty} \frac{\sin \upsilon}{\upsilon}\, d\upsilon \right\}\right] \tag{9}
$$

- where $\upsilon = 2\kappa r_0$. The integrals with respect to both υ and z are readily obtainable from tables. It is perhaps worth emphasising that this comparatively straightforward expression for a quantity normally apprehended by much more sophisticated methods is made possible by using the simple plane-wave pressure-velocity relation to obtain source velocity. This accordingly restricts the use of the method to values of κa not less than 0.75, the upper limit being 2, as indicated earlier. Some justification for putting forward a method covering a limited range of piston sizes is the fortunate circumstance that transducers currently under development at A.R.L. are comfortably within this range. The possibility of extending the range of κa values will be studied. At the lower end, i.e. $\kappa a < 0.75$, the good approximation to a rigid piston given by a hemispherical source might be exploited, whilst values exceeding 2 will involve the consideration of additional terms in the series (2).

There is, as far as the writer is aware, rather little reliable data available with which the results of the present method may be compared. The work of S. Hanish[2] provides values of self resistance and reactance derived by much more advanced methods, but which apply only to the completely unbaffled condition and then only for values of κa not exceeding unity. The unbaffled condition is given in the present method by putting $b = a$; that is, by making the lower limit of the integrals with respect to υ equal to $2\kappa a$ in (9). For example, putting $\kappa a = 1, b = a$, and dividing (9) by $\pi a^2 u \rho c$ to give specific impedance, and finally deducting the result from the known infinitely-baffled self-impedance value, the value for the unbaffled condition becomes $Z = 0.2372 + i\,0.6108$ as compared with $0.2322 + i\,0.5812$ by Hanish. Repeating this with $\kappa a = 0.75$ gives $Z = 0.136 + i\,0.4633$ against Hanish's $0.1337 + i\,0.4512$. Below $\kappa a = 0.75$ all resemblance between the present results and those of Hanish rapidly disappears, the latter being evidently above suspicion if only because at those sizes they should and do conform to the simple spherical source equivalent.

Since the unbaffled condition is given here merely by using one particular value of κb in a general expression, the agreement shown above permits some confidence in the results for partial baffling. The latter are shown graphically in Fig. 2 for $\kappa a = 0.75$, 1 and 2 and are in agreement

with some results given by Nimura and Watanabe[3] in showing that certain finite sizes of baffle produce values of self resistance and reactance greater than those obtained with an infinite baffle.

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PHGC/EH

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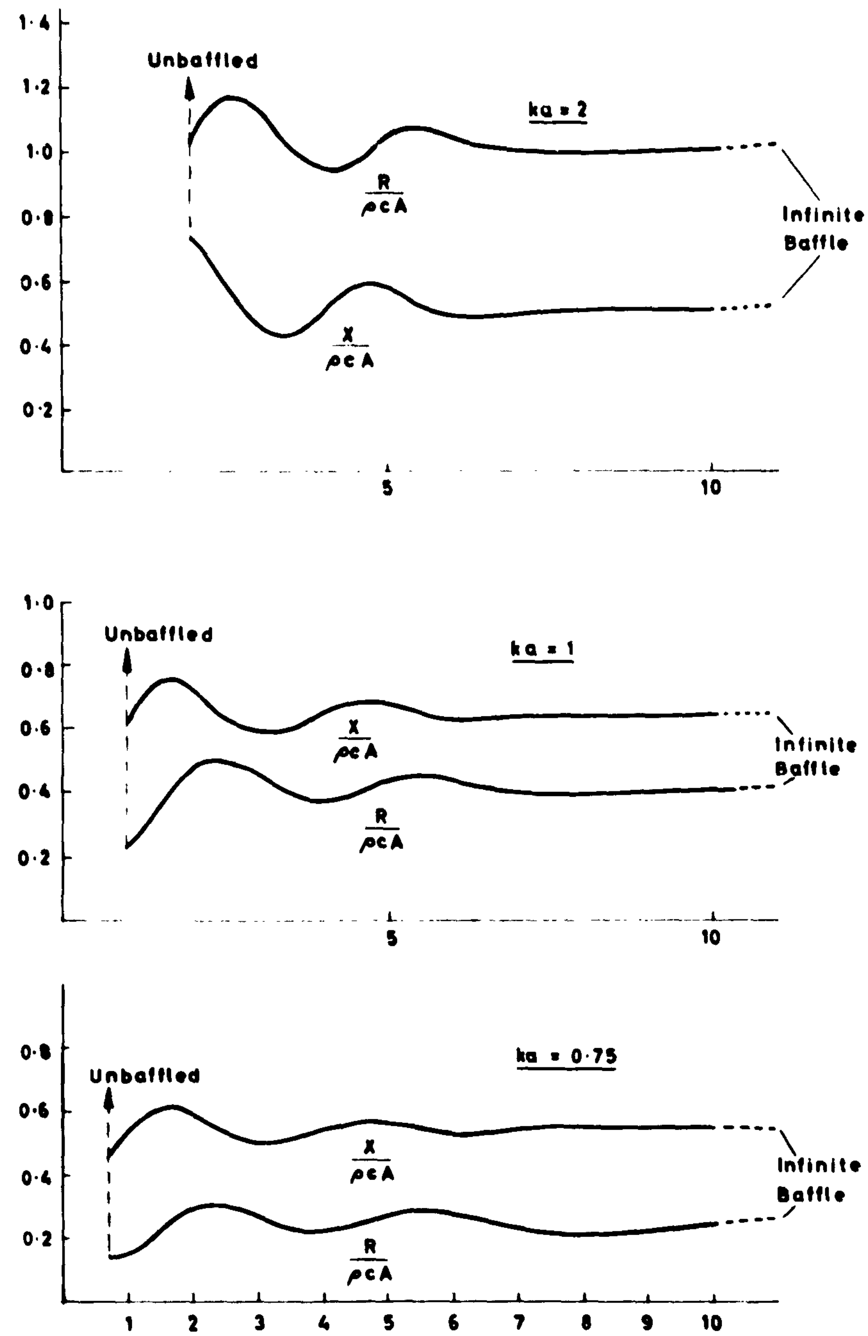

a = Piston Radius ; b = Baffle Radius.

FIG. 2 VARIATION OF ACOUSTIC SELF IMPEDANCE WITH BAFFLE RADIUS

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